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EFFECT OF BLADE COOLING ON PERFORMANCE OF A GAS TURBINE POWER PLANT

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(1950)

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May 23, 1955

Thesis
B93

EFFECT OF BLADE COOLING ON PERFORMANCE OF A GAS TURBINE POWER PLANT

by

Bernard L. Buteau

(Submitted to the Department of Naval Architecture on May 23, 1955, in partial fulfillment of the requirements for the degree of Naval Engineer in Naval Architecture.)

ABSTRACT

This report is an analytical study of the effects on specific air consumption and thermal efficiency of three types of blade cooling systems. The methods considered are:

(1) Air Forced Convection Cooling
(2) Air Transpiration Cooling
(3) Liquid Forced Convection Cooling

The analysis was applied to a simple gas turbine powerplant containing a cooled turbine and an uncooled turbine, both of which were composed of an infinite number of infinitesimal stages. Certain general design parameters were chosen for each coolant system and the effect of variation of these parameters on powerplant performance was studied. The three cooling methods were also compared against one another for otherwise identical conditions in the basic powerplant.

The results of this work make it possible to predict, in a semi-quantitative manner, the effect of cooling system design on powerplant performance. In particular, the results of this report show that for well designed cooling systems operating at design conditions, appreciable net gains in performance can be realized by increasing turbine inlet temperature beyond the temperature which the uncooled blades can withstand.

Thesis Supervisor: Warren M. Rohsenow

Title: Associate Professor of Mechanical Engineering

28914

Mass. Inst. of Tech.
Cambridge 39, Mass.
May 23, 1955

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

I hereby submit a thesis entitled, "Effect of Blade Cooling on Performance of A Gas Turbine Power Plant," in partial fulfillment of the requirements for the degree of Naval Engineer in the Department of Naval Architecture at the Massachusetts Institute of Technology.

Respectfully yours,

Bernard L. Buteau
Lieutenant (junior grade)
U. S. Navy

ACKNOWLEDGMENTS

The initial work in this study was undertaken by Mr. James C. Burke, a research assistant with the Mechanical Engineering Department, Massachusetts Institute of Technology. This thesis represents an extension and evaluation of the analysis developed by him.

The author is especially grateful for the patient cooperation and advice offered by his thesis supervisor, Professor Warren M. Rohsenow.

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I. INTRODUCTION

Many schemes have been devised for cooling gas turbine blades. Most involve circulating a coolant (air, water or fuel) through passages in the blades either as part of a closed circuit externally cooled or as a "one way" process whereby the coolant discharges into the turbine working fluid after having absorbed heat from the blades.

All of these methods, have, in addition to the functional design problems, the disadvantage of adding losses to the powerplant which, in part, reduces the gains to be anticipated from increased turbine inlet temperature.

In M.I.T. Technical Report No. 4 (reference 14), the performance of a simple gas turbine powerplant utilizing cooled blades was analyzed for a wide range of operating conditions. This analysis considered only the effect of heat loss from the working fluid to the cooled blades and did not investigate the losses which might be incurred in circulating the coolant. In this study the above analysis is extended to include the over-all effects on powerplant efficiency and specific air consumption of three types of cooling systems.

1) Forced convection air cooling -- air is pumped through radial passages in the blades and discharges into the working fluid.

2) Transpiration air cooling -- air is forced through hollow porous blades and discharges into the working fluid.

3) Liquid cooling -- liquid is circulated through passages in the blades in a closed circuit and cooled externally.

II. FORCED CONVECTION -- AIR COOLED POWERPLANT

A. Description of System

The basic powerplant studied in this investigation is composed of a compressor (C_1), a burner (D), a cooled turbine (A) and an uncooled turbine (B). Each turbine is composed of an infinite number of infinitesimal impulse stages of polytropic efficiency η_p. The coolant air is supplied by a bleed connection from compressor, C_1. The expansion ratio of turbine A varies so that the inlet temperature to turbine B remains constant at the maximum allowable blade temperature. The coolant (m) is compressed in C_1, passes through a heat exchanger (E) and is ducted to coolant passages leading to the buckets and nozzles. Appropriate portions of the coolant, m_N and m_B emerge from the nozzles and buckets, and, at states 6e and 6d respectively, mix with the main working fluid in the mixing chamber (F). The resultant mixture, now at state 7, expands through the uncooled turbine B.

The assumption that the coolant does not mix with the working fluid within the cooled turbine A can be visualized physically as follows: The coolant could be thought to discharge into the cooled turbine but to travel unmixed along the periphery of the casing. This viewpoint assumes that any work done by the coolant within turbine A would be counterbalanced by the aerodynamic losses which it causes. According to this hypothesis, the coolant would discharge against local turbine pressure -- which for the limiting case would be the inlet pressure of turbine A.

In this powerplant it is seen that, in addition to the heat loss to the blades, other losses are present such as: (1) the work

input to the coolant in the compressor, (2) the pumping work done on the coolant as it passes through the rotor and blades, and (3) the enthalpy loss of the main working fluid due to mixing with the coolant at F. However, to some extent, energy will be recovered when the coolant expands through the uncooled turbine B.

The two design parameters listed below were chosen as being important to the cooling system and its effect on the over-all power-plant. The base value represents the value at which each was fixed when either of the other two parameters was varied.

	Design Parameter	Base Value	Range Studied
η_x	- coolant passage effectiveness	0.6	0.4 - 0.8
T_{o3b}	- coolant inlet temperature	1200°R	600°R - 1500°R

η_x is a measure of the heat transfer from blade to coolant (and, from heat transfer — friction relationships, a measure of blade pressure drop). η_x is defined as

$$\eta_x = \frac{T_{o5e} - T_{o4e}}{T_w - T_{o4e}} \quad \cdots\cdots \quad (1)$$

As shown above, $_x$ may be written immediately in terms of nozzle coolant temperatures but the effect of centrifugal temperature rises must be considered (see Appendix F) in order to relate η_x to bucket coolant temperatures. Also, in Appendix F, it is shown that η_x is a function of coolant passage ℓ/D_e ratio and, to a lesser extent, coolant Reynold's number.

T_{o3b} is the temperature of the coolant air leaving the heat exchanger E and may be thought of as representing the heat absorbing

potential of the coolant. The coolant temperature in this analysis is maintained independent of the outlet conditions of the auxiliary compressor by the heat exchanger E. The powerplant is not, however, charged with the heat added or subtracted from the coolant in E. The base value of T_{o3b}, 1200° R, corresponds to what would be the available coolant temperature if the coolant air were bled from the top of the main compressor without passing through the heat exchanger.

B. Method of Analysis -- Forced Convection Air Cooled Powerplant

The general expression for the efficiency of a heat engine is

$$\eta = \frac{W_{net}}{Q_{in}} \quad \ldots\ldots \quad (2)$$

Referring to Fig. 1, the efficiency of the forced convection air cooled powerplant may be written as follows:

$$\eta = \eta_M + m/M\,\eta_m \quad \ldots\ldots \quad (3)$$

where

$$\eta_M = \frac{W_A + W_B - W_{C_1}}{Q_{in}} \quad \ldots\ldots \quad (4)$$

and

$$\eta_m = \frac{W_B - W_p - W_{C_1}}{Q_{in}} \quad \ldots\ldots \quad (5)$$

The term W_p in Equation (5) is the pumping work done by the rotor on the coolant flowing within the rotor.

As shown in Appendix D, η_M, the efficiency of the main working fluid cycle, may be determined by the method outlined in

reference 18 if the values of Q_1/W_A and $\Delta H/W_A$ are known. Q_1 represents the heat loss from the working fluid to the cooled blades. ΔH represents the enthalpy loss to the main working fluid caused by mixing with the coolant. The analysis used to evaluate Q_1/W_A and $\Delta H/W_A$ is described below.

Appendix E presents a relation for the heat flow to a cascade of blades and an optimum design value for the work of an infinitesimal stage. These two relations can be combined as per Equation (6) below. To obtain average values of this ratio for the nozzle or bucket rows, T_o should be interpretated as the average stagnation temperature relative to either nozzles or buckets. Making use of the average stagnation temperatures (Equations 7 and 8) and noting, from Fig. 3a, that $T_{o4a} = T_w + \Delta H/C_{p_2}$, expressions for $(Q_1/W_A)_N$, $(Q_1/W_A)_B$ and Q_1/W_A are determined.

$$\frac{Q}{W} = \frac{C_p(T_o - T_w)(e^{\xi_o/2} - 1)}{K_1 \frac{u^2}{2g_oJ}} \quad \ldots\ldots \quad (6)$$

$$(\overline{T}_{or})_N = \frac{T_{o3a} + T_{o4a}}{2} \quad \ldots\ldots \quad (7)$$

$$(\overline{T}_{or})_B = \frac{T_{o3a} + T_{o4a} - \frac{u^2}{2C_pJg_o}}{2} \quad \ldots\ldots \quad (8)$$

$$\left(\frac{Q_1}{W_A}\right)_N = \frac{C_{p_1}\left(T_{o3a} - T_w + \frac{\Delta H}{C_{p_2}}\right)(e^{\xi_o/2} - 1)}{2K_1 \frac{u^2}{2g_oJ}} \quad \ldots\ldots \quad (9)$$

$$\left(\frac{Q_1}{W_A}\right)_B = \frac{C_{p_1}\left(T_{o3a} - T_w + \frac{\Delta H}{C_{p_2}} - \frac{u^2}{C_p J g_o}\right)\left(e^{\xi_o/2} - 1\right)}{2K_1 \frac{u^2}{2g_o J}} \quad \ldots\ldots \quad (10)$$

$$\frac{Q_1}{W_A} = \left(\frac{Q_1}{W_A}\right)_N + \left(\frac{Q_1}{W_A}\right)_B \quad \ldots\ldots\ldots\ldots \quad (11)$$

$\Delta H/W_A$, the enthalpy loss term, is found by applying an energy balance to the adiabatic mixing at F. This derivation is carried out in Appendix G and results in Equation (12). The terms ϵ and σ, which are defined and tabulated in Appendix G, account for temperature rises due to rotational work. The term η_x' (see Appendix F) is an apparent effectiveness which includes rotational effects as well as heat transfer.

$$\frac{\Delta H}{W_A} = \frac{C_{p_2}}{C_{p_3}} \left(\frac{Q_1}{W_A}\right)_N \left(\frac{1 - \eta_x}{\eta_x}\right) + \left(\frac{Q_1}{W_A}\right)_B \left(\frac{1 - \eta_x' - \epsilon}{\eta_x' - \sigma}\right) \quad \ldots \quad (12)$$

The above Equations (9 - 12) have been solved by iteration for Q_1/W_A and $\Delta H/W_A$, employing the additional relation that

$$\frac{\Delta H}{C_{p_2}} = \frac{\Delta H}{W_A} \cdot \frac{W_A}{C_{p_2}} = \frac{\Delta H}{W_A}\left(\frac{1}{C_{p_2}}\right)\left(\frac{h_{o3a} - h_{o7}}{1 + \frac{Q_1}{W_A} + \frac{\Delta H}{W_A}}\right)$$

The values of η_N obtained are plotted in Fig. 5.

m/M, the total coolant ratio required to maintain the blades in the cooled turbine at T_w is obtained by applying energy balances to the coolant flows in the buckets and nozzles (see Fig. 4a). Implicit in this analysis, is the assumption of infinite blade thermal conductivity (i.e., cooled blade temperature is everywhere constant and equal to T_w).

The coolant ratio for the nozzles is determined from a simple heat balance and the definition of effectiveness (Equation (1).

$$(m/M)_N = \frac{(Q_1/W_A)_N W_A}{C_{p_3}\eta_x(T_w - T_{o3b})} \qquad \cdots\cdots \qquad (13)$$

The required coolant ratio for the buckets is found by the same general technique but the details are complicated by centrifugal effects. The derivation is carried out in Appendix E.

$$(m/M)_B = \frac{(Q_1/W_A)_B W_A}{C_{p_3}\left\{\eta_x'\left[T_w - T_{o3b} - \frac{u^2}{2C_pJg_o}\left(\frac{r_h}{r_T}\right)^2\right] - \frac{u^2}{2C_pJg_o}\left[1 - \left(\frac{r_h}{r_T}\right)^2\right]\right\}} \qquad (14)$$

The total required coolant ratio, the sum of Equations (13) and (14), is plotted in Fig. 6 as a function of turbine inlet temperature and the cooling system design parameters.

The quantity η_m is shown in Equation (5) to depend on W_B, Q_{in}, W_p and W_{C_2}. W_B and Q_{in} are determined by the methods of Appendix D. W_p, the pumping work, is expressed by the following equation derived in Appendix H.

$$W_p = \frac{(m/M)_B}{m/M}\ \frac{u^2}{g_oJ} \qquad \cdots\cdots \qquad (15)$$

The work done by compressor C on the coolant is a function of both the net stagnation pressure drop in the coolant flow path and the stagnation pressure at the blade tips, p_{o6b}. For simplicity, the pressure drops in both nozzles and buckets are considered to be similar. It is further assumed that the stagnation pressure at the

turbine inlet nozzle tips is low enough to permit positive flow from the compressor bleed point. This reasoning permits the use of a single compressor bleed connection. This study assumes that all of the coolant will be bled at compressor discharge pressure and the correct distribution of coolant will be maintained by throttling the paths of lower resistance. This assumption furnishes a conservative estimate of the compressor work accomplished in the coolant cycle.

η_m, as calculated in accordance with the preceding discussion is plotted in Fig. 5. Now, in conjunction with the values of η_M and m/M previously obtained it is possible to determine η per Equation (2). The powerplant efficiency, η, and specific air consumption are plotted in Figs. 7-8. In this investigation specific air consumption is based on total air flow as follows:

$$\text{specific air consumption } \left(\frac{lb_m}{\text{hp-hr}}\right) = \frac{(2545)(1 + m/M)}{(\eta)\,(Q_{in})} \qquad (16)$$

C. Results of Analysis -- Forced Convection

In Fig. 5, η_M and η_m are plotted against turbine inlet temperature for a range of cooling system design parameters. It is seen that η_M, the efficiency of the main working fluid cycle, is primarily a function of η_x and turbine inlet temperature. This is because, at constant T_{o3a}, changes in η_M result from variations in the mixing loss $\Delta H/W_A$. $\Delta H/W_A$ is relatively unaffected by coolant inlet temperature since, say, increasing T_{o3b} tends to both increase coolant flow and increase coolant discharge temperature. Both of these effects work in the opposite direction and tend to be mutually cancelling.

η_m, which may be thought of as a measure of the efficiency of the coolant cycle is, to a first approximation, a function of η_x and T_{o3a}. This is because, at constant T_{o3a}, changes in η_m result primarily from changes in W_B. These changes in W_B are the result of variations in the mixing loss $\Delta H/W_A$, which is relatively unaffected by changes in T_{o3b}.

The coolant ratios required at various turbine inlet temperatures are plotted in Fig. 6. As might be expected, increasing η_x and reducing T_{o3b} help to minimize the coolant ratio. The effect of coolant inlet temperature is especially marked. At $T_{o3a} = 3000°R$, decreasing the coolant inlet temperature from 1500°R to 500°R reduces the required coolant ratio from 71% to 18% for $\xi_o = 0.08$. For $\xi_o = 0.02$ the coolant ratios are much less.

In Fig. 7, the effect of turbine inlet temperature and coolant passage effectiveness on powerplant design point performance is shown. Throughout the range of turbine inlet temperature studied, increasing η_x improved thermal efficiency, and lowered specific air consumption. The gains arising from increasing η_x are due to the increases in η_M and decreases in m/M which result. The greatest gains in performance appear to lie in eliminating very low values of η_x rather than in attaining near 100% effectiveness.

The effect of coolant inlet temperature on powerplant performance is shown in Fig. 8. T_{o3b} is found to have little effect on maximum efficiency, but reducing T_{o3b} does result in much lower minimum specific air consumption and much better efficiency at high

turbine inlet temperatures. The beneficial effects of reducing T_{o3b} are due mainly to the decreased coolant ratios required.

The results of Figs. 7 and 8 show that the design point performance of a forced convection air cooled powerplant may be optimized by increasing blade coolant passage effectiveness, and reducing coolant inlet temperature. For the particular operating conditions studied, η_x and T_{o3b} have more pronounced effects on specific air consumption than on maximum cycle efficiency.

If metallurgical considerations did not exist, it would be possible, by increasing the turbine inlet temperature of a similar uncooled powerplant from 1960°R to 3000°R to increase cycle efficiency from 35% to 41% and reduce specific air consumption from 37 to 13 lb_m/hp-hr. (See Fig. 18). In the forced convection air cooled powerplant, in which the blade temperature is limited to 1960°R, if the turbine inlet temperature is increased through the same range, depending on the values of the cooling system design parameters, the cycle efficiency can be increased 2.5% to 3.5% (out of a possible 6% gain) and the specific air consumption can be lowered 13 to 18 lb_m/hp-hr (out of a possible 25 lb_m/hp-hr reduction). From these results it is apparent both that (1) appreciable net improvements in performance can be realized with forced convection air cooling and (2) the cooling system design parameters chosen in this analysis have a significant effect on powerplant performance.

III. TRANSPIRATION COOLED POWERPLANT

A. Description of System

The basic transpiration air cooled powerplant is similar to that described in I-A. The difference lies in the manner in which the coolant passes through the blades and mixes with the working fluid.

As shown in Fig. 2, the main working fluid (M) expands through the cooled turbine A, mixing with the coolant which has transpired through the cooled blades. Appropriate portions of the coolant, m_N and m_B, are forced through the nozzles and buckets respectively, enter the cooled turbine at states 6e and 6d, and mix with the main working fluid. The mixture of coolant and main working fluid leaves the cooled turbine at state 7 and is expanded through a second uncooled turbine B.

It is assumed that the addition of the coolant mass has no net effect on the expansion process within turbine A. That is, the work done by the coolant mass is exactly counterbalanced by the enthalpy losses due to mixing and the aerodynamic losses due to boundary layer thickening.

In general, the losses for the transpiration cooled powerplant are similar in nature to those in the forced convection cooled powerplant. Two major differences exist however: (1) As mentioned above, it is assumed there is no net enthalpy loss due to mixing of the coolant and working fluid (i.e., $\Delta H/W_A = 0$). (2) The heat flow to the blades, Q_1, will be materially reduced due to the insulating film produced by the transpiring coolant.

The design parameter governing the operation of the transpiration cooling system is listed below. The parameter η_x used in the forced convection system is now dropped because the contact between coolant and wall is so great that no significant temperature difference exists between the emerging coolant and the blade wall.

Design Parameter	Base Value	Range
T_{o3b} coolant inlet temperature	1200° R	600-1500

T_{o3b} has approximately the same effect on the transpiration system as on the forced convection system. The discussion in I-A for T_{o3b} also applies here.

B. Method of Analysis -- Transpiration

The efficiency of the transpiration cooled powerplant may be expressed by Equations 3, 4, and 5. However, because of the differences in the physical processes the determination of the component terms of these equations is different from the methods developed in I-B.

The calculation of η_M for given values of Q_1/W_A is similar to the method for forced convection. The details are outlined in Appendix D. As will be shown below, however, the term Q_1/W_A can no longer be evaluated solely by the equations developed in II-A.

In a transpiration cooled powerplant, the coolant forms an insulating layer which reduces the gas-to-blade heat transfer coefficient, α . Allowing starred terms to denote heat transfer conditions for zero transpiration mass flow, the following equations are written.

$$(Q_1/W_A)_N = (Q_1/W_A)_N^* \, (\alpha/\alpha^*)_N \qquad \ldots\ldots \qquad (17)$$

$$(Q_1/W_A)_B = (Q_1/W_A)_B^* \, (\alpha/\alpha^*)_B \qquad \ldots\ldots \qquad (18)$$

Now $(Q_1/W_A)_N^*$ and $(Q_1/W_A)_B^*$ can be evaluated from Equations (9) and (10). (Noting that, for transpiration, $\Delta H/C_{p_2} = 0$).

Several methods are given in the literature for determining the ratio α/α^*. (References 4 and 5). A simple relation is the "film theory" expression proposed by Mickley (reference 13).

$$\alpha/\alpha^* = \frac{\dfrac{G_t C_p}{\alpha^*}}{e^{\left[\dfrac{G_t C_p}{\alpha^*}\right]} - 1} \qquad \ldots\ldots \qquad (19)$$

In Appendix I, Equation (19) is rewritten in terms of the nozzle coolant ratio and the loss factor ξ_o. This equation applies equally well for the buckets if $(\alpha/\alpha^*)_N$ and $(m/M)_N$ are replaced by $(\alpha/\alpha^*)_B$ and $(m/M)_B$.

$$(\alpha/\alpha^*)_N = \frac{(m/M)_N/(e^{\xi_o^*/2} - 1)}{e^{\left[\dfrac{(m/M)_N}{e^{\xi_o^*/2} - 1}\right]} - 1} \qquad \ldots\ldots \qquad (20)$$

The coolant ratio for the nozzles is determined from a simple energy balance applied to the cooling process (see Fig. 4b). It is assumed in this energy balance that the coolant leaves the blade wall at wall temperature.

$$(m/M)_N = \frac{(Q_1/W_A)_N \, W_A}{C_{p_3} \, (T_w - T_{o3b})} \qquad \ldots\ldots \qquad (21)$$

The coolant ratio for the buckets is obtained in a similar manner. The energy balance is applied at a mean blade radius such that the blade tangential velocity is $\bar{u} = u(\frac{r_T + r_h}{2})$. Therefore, the absolute stagnation temperatures of the coolant before and after the porous wall are $T_{o5d} = T_{o3b} + \frac{\bar{u}^2}{2C_p J g_o}$, and $T_{o6d} = T_w + \frac{\bar{u}^2}{2C_p J g_o}$.

$$(m/M)_B = \frac{(Q_l/W_A)_B\, W_A}{C_{p_3}(T_w - T_{o3b} - \frac{\bar{u}^2}{2C_p J g_o})} \quad \ldots\ldots \quad (22)$$

From Equations (17) through (22), the coolant ratios and heat loss terms (Q_l/W_A) are calculated. The resulting curves of ${}_M$ and m/M are plotted in Figs. 9 and 10.

${}_m$, is determined by evaluating W_B, Q_{in}, W_p, and W_{C_1}. W_{C_1}, W_B and Q_{in} are calculated as shown in Appendix D. An expression for W_p, the rotor pumping work is derived in Appendix H, assuming that the rate of transpiration per unit radial length is constant in the buckets.

$$W_p = .730\,\frac{(m/M)_B}{m/M}\;\frac{u^2}{g_o J} \quad \ldots\ldots \quad (23)$$

C. The Results of Analysis -- Transpiration Cooling

In Fig. 9, η_M and η_m are plotted against turbine inlet temperature for various constant values of the cooling system design parameter. It is seen that increasing coolant inlet temperature causes a small increase in the quantity η_M. This is because higher values of T_{o3b} result in larger coolant flows which in turn increase the coolant insulating effect and reduce heat loss to the blades.

The required coolant ratios are plotted in Fig. 10. As in the case of the forced convection cooling system, increasing T_{o3b} increases the coolant ratio. The most important feature of these curves is the low coolant ratios required for transpiration as compared to those required for forced convection. When $T_{o3b} = 1200^\circ$ R and $T_{o3a} = 3000^\circ$ R, the coolant ratio required for transpiration is less than one third of that required for forced convection.

In Fig. 11 the effects of the design parameter T_{o3b} on powerplant performance is shown. As in the case of forced convection reducing coolant inlet temperature tends to increase efficiency and lower specific air consumption. However, the transpiration cooled powerplant is much less sensitive to variation in T_{o3b} than was the forced convection powerplant.

IV. LIQUID COOLED POWERPLANT

The liquid cooled powerplant considered in this study is cooled by a closed water circuit. The water flows through loop type passages in the blades and is cooled externally in a heat exchanger. If the coolant flows through both nozzles and buckets in series, the analysis of Appendix M shows that, for a wide range of operating variables, the net coolant pressure rise in the buckets and rotors is sufficient to overcome all the system frictional pressure drops and no circulating pump is necessary. Also, since the coolant flows both outward and inward in the rotor and buckets no net pumping work is done by the rotor.

From the discussion above, it is seen that the quantity η_m (Equation 5) is zero and that η is just equal to η_M. The cycle efficiency for the water cooled powerplant is then calculated according to the methods of Appendix D. The results of these calculations are plotted in Fig. 12.

V. SURVEY OF BLADE COOLING ANALYSES

Currently much research is going into the development of high temperature blade alloys and methods of blade cooling. Considerable work has been devoted to the design of particular types of coolant systems, but relatively little analytical work has been done on the thermodynamic performance of cooled turbines. A survey of the technical literature reveals a large amount of material on the subject of blade cooling. However, most of these studies are concerned with the blade temperature distribution and blade Nusselt number (references 6, 9, 11, 16). The survey will be discussed within areas of development common to most of the analyses.

A. Heat Transfer Correlations

Outside surface heat transfer coefficients have been obtained experimentally by many investigators for static cascades of turbine blades. The resulting correlation equations have been of the form

$$Nu = A\ (Re)^B\ (Pr)^C$$

However, each investigator has used a different set of fluid conditions in defining the nondimensional parameters. Consequently, the constants A, B, and C vary with each analysis. Ainley (reference 1)

stated that from tests of an experimental air-cooled turbine there was a clear inference that heat transfer coefficients, derived from cascade tests, were not necessarily a reliable guide to heat transfer on similar blades in a turbine stage. He explained that the differences were probably associated with differences in main stream turbulence and secondary flows.

Several theoretical heat transfer correlations based on Reynolds' analogy have been developed. Brown and Donoughe (reference 3) extended the boundary layer heat transfer theory to obtain

$$\frac{\alpha}{C_p G} (Pr)^{2/3} = \frac{f}{2}$$

Smith (reference 16) used a modified form of Reynolds' analogy to determine the blade Nusselt number. The modification consisted of an empirically determined factor which varied with the ratio of exit to entry velocity. This factor was only good at a Reynolds' number of about 2 x 10^5 but could presumably be extended. Smith noted discrepancies between experimental and theoretically calculated Nusselt numbers. He attributed the disagreement to variations in blade shape rather than to errors in the theoretical analysis. Hawthorne (reference 8) developed a similar correlation which was used in the analysis presented in this study (Appendix B). By using Reynolds' analogy, he was able to relate the heat transfer coefficient to the profile loss factor ξ_p as determined in cascade tests. As of this writing, there has been no good experimental data obtained from rotating cooled turbine blades to correlate the analyses presented here.

B. Turbine Cooling Losses

All of the proposed methods of blade cooling for gas turbines have the disadvantage of adding losses to the systems which reduce the gains to be anticipated from increased turbine inlet temperatures. The losses incurred due to cooling are as follows:

a) Aerodynamic losses caused by larger tip clearances and flow interference in the regions where the coolant is discharged into the main gas stream.

b) Cooling losses caused by heat transfer from working fluid to coolant which results in a reduction of the reheat factor due to progressively lower gas temperatures in the cooled turbine as compared to the uncooled turbine.

c) Mixing losses which result in a lowering of the cooled turbine exhaust temperature and decrease the performance of either a heat exchanger or a second uncooled turbine as in this study.

C. Aerodynamic Losses

Hawthorne (reference 8) showed that the aerodynamic losses were of the order of 1 or 2 percent, changing slightly with Mach number and type of stage. By averaging a large number of tests, Ainley (reference 1) concluded that the combined effect of nozzle and rotor cooling air quantities was to reduce stage efficiency by less than 0.5 percent when the cooling flow ratio totalled 4 percent. This is seen to be less than the effect predicted by Hawthorne, but it is no doubt due to the low cooling flow ratio employed.

D. Cooling Losses

Brown (reference 2) developed an analysis to determine the effect of cooling on turbine efficiency and reheat factors. He derived a cooling loss factor

$$\ell = C \frac{AB}{d^{0.5} u^{2.5}} (T_o - T_w)$$

where A was a function of the gas properties, B a function of blade properties, and C a constant. This loss factor was then used to obtain the over-all turbine efficiency.

$$T = \frac{1}{1 + \ell} \quad \frac{1 - \left(\frac{1}{r_A}\right)^{\frac{K-1}{K} \eta_p (1+\ell)}}{1 - \left(\frac{1}{r_A}\right)^{\frac{K-1}{K}}}$$

This expression is identical with that determined by Hawthorne and used in this study (Appendix D). The variation arises in the derivation of ℓ or Q/W as used in this analysis. Brown's loss factor does not account for the decrease in relative stagnation temperature resulting from blade rotation while Q/W does (Equation 10). However, the loss factor ℓ does bring out the fundamental relation between cooling loss and the variables which determine turbine blade design (i.e., variables in the quantities A and B). Therefore, ℓ does tend to indicate the way to optimum design. Hawthorne's factor Q/W (Equations 9 and 10) does not accomplish this purpose since the variables which determine blade design are hidden in the empirical loss factor ξ_o (Appendix E). Esgar and Ziemer (reference 7) produced a

series of charts which permitted the graphical evaluation of cooled turbine performance. They assume that all of the heat rejection occurs at turbine inlet temperature while this study assumes a mean temperature between turbine inlet and exhaust for calculating heat rejection rates. Heat rejection rates calculated by Hawthorne and by Brown show surprisingly good agreement while those of Esgar and Zeimer are from one-third to one-half those calculated by references 8 and 14.

E. Mixing Losses

The mixing loss is represented by $\Delta H/W$ (Appendix F) in this study. This loss was not calculated or included in any other analysis covered by this survey. However, several of the analyses (references 2, 7, 15) indicated the need for including such losses. The assumptions used in deriving $\Delta H/W$ (see Section II-A) were identical with those suggested by the NACA (references 7, 15) for calculation of the mixing losses. (See assumption D below.)

F. Assumptions

In order to obtain simplified analyses each investigator was required to make many assumptions. The major assumptions are listed here for purposes of comparison with this study.

a) The gas temperature profile was assumed to be uniform. (References 2, 15)

b) Blade geometry, and inside and outside heat transfer coefficients were assumed constant over the span of the blade. (References 2, 15)

c) Radiation and conduction effects were neglected. (References 2, 15)

d) The coolant was not considered to be part of the working fluid after it was discharged into the cooled turbine. The coolant mixed with the main working fluid at the exhaust condition from the cooled turbine. (References 7, 15)

e) The entire velocity head of the cooling air was lost or counterbalanced by the aerodynamic losses imposed in the cooled turbine. (References 7, 15)

VI. CONCLUSIONS

1. The results of this investigation show that appreciable net improvements in gas turbine performance can be attained by the companion processes of increasing turbine inlet temperature and blade cooling.

2. Liquid cooled turbines are capable of lower specific air consumption than air cooled turbines.

3. Transpiration air cooling results in lower specific air consumption and less sensitivity to cooling system design parameters than does forced convection cooling.

4. For both types of air cooled powerplants, decreasing coolant inlet temperature and reducing coolant discharge pressure improved powerplant performance. For the forced convection system, blade internal geometry (i.e., coolant passage effectiveness) has significant effects on performance.

5. The blade loss factor ξ_o has a noticeable effect on plant performance. Decreasing ξ_o increases efficiency, decreases specific air consumption, and increases the maximum inlet temperature at which peak efficiency occurs.

6. Low specific air consumption indicates higher specific output for smaller plants. For this reason, transpiration cooled gas turbines would tend to be more desirable than convection cooled turbines for aircraft applications.

7. High efficiencies indicate better specific fuel consumption. Because of this and an abundance of water, the liquid cooled gas turbine will be more attractive for marine installations.

8. The assumptions, used in this analysis and in the analyses discussed in Section V, yield results which are adequate for comparative purposes. This study shows good agreement with reference 2; however, it is at variance with work done by the NACA (reference 7). Experimental data to prove or disprove these analyses is not available at the present time.

VII. ILLUSTRATIONS

Fig. Number	Caption
1	Schematic of gas turbine powerplant with forced convection air cooled blades
2	Schematic of gas turbine powerplant with transpiration air cooled blades
3	Typical cycle diagrams for main working fluid, M
4	Energy balances applied to cooling processes
5	Variation in η_M and η_m with turbine inlet temperature and cooling system design parameters for forced convection air cooled powerplant
6	Effect of turbine inlet temperature and cooling system design parameters on required coolant ratio for forced convection air cooled powerplant
7	Effect of blade coolant passage effectiveness on performance of forced convection air cooled powerplant
8	Effect of coolant inlet temperature on performance of forced convection air cooled powerplant
9	Variation in η_M and η_m with turbine inlet temperature and cooling system design parameters for transpiration air cooled powerplant
10	Effect of turbine inlet temperature and cooling system design parameters on required coolant ratio for transpiration air cooled powerplant
11	Effect of coolant inlet temperature on performance of transpiration air cooled powerplant
12	Comparison of performance of powerplants employing different systems of cooling

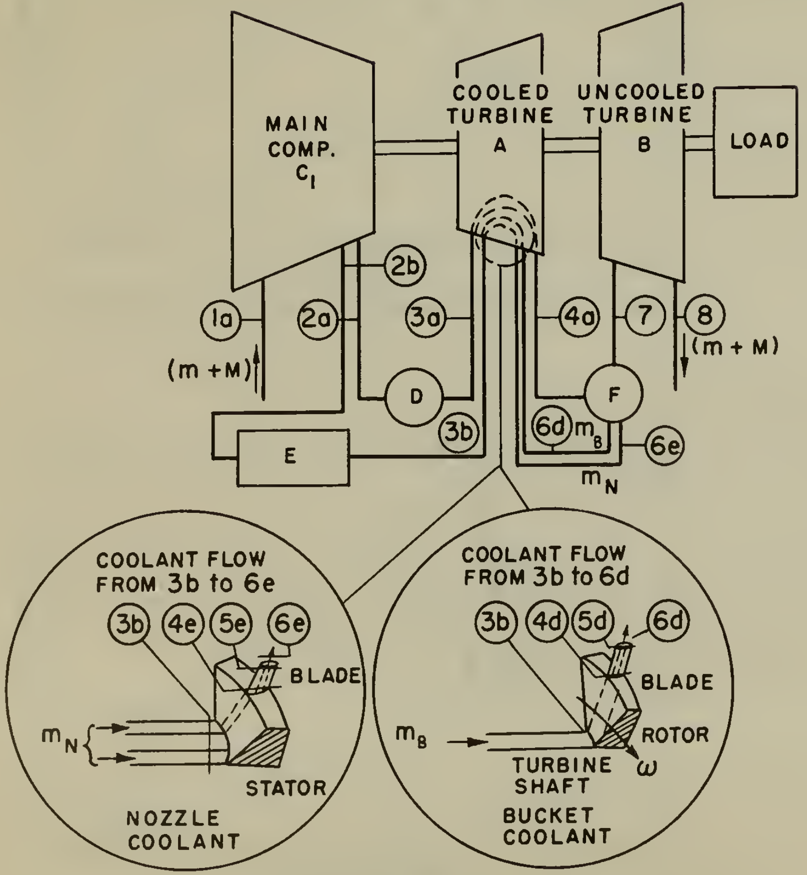


FIG. 1 – SCHEMATIC OF GAS TURBINE POWER PLANT WITH FORCED CONVECTION AIR COOLED BLADES

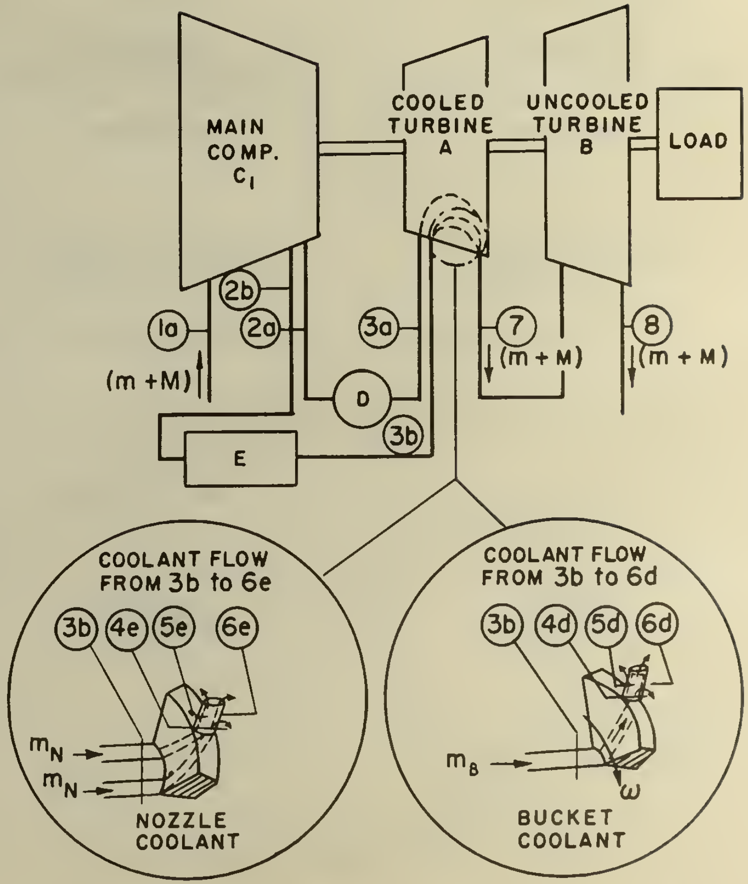
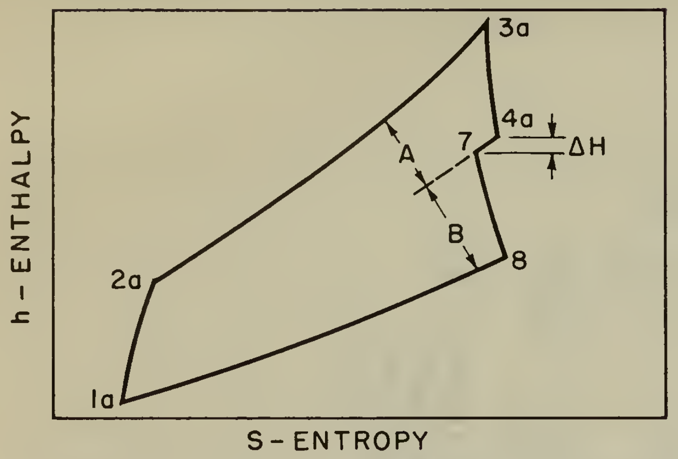
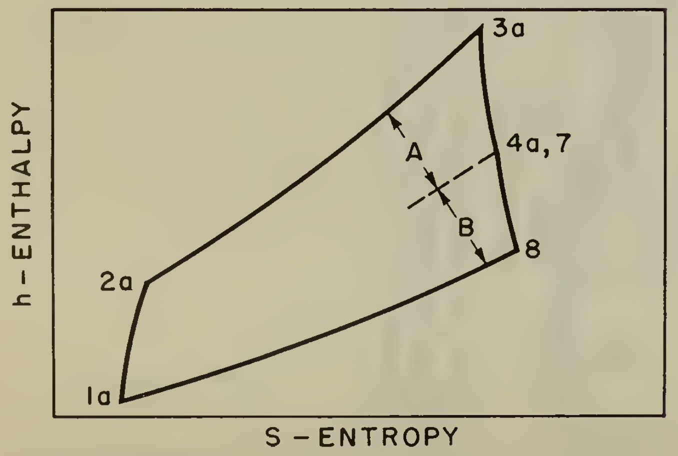


FIG. 2 – SCHEMATIC OF GAS TURBINE POWER PLANT WITH TRANSPIRATION AIR COOLED BLADES

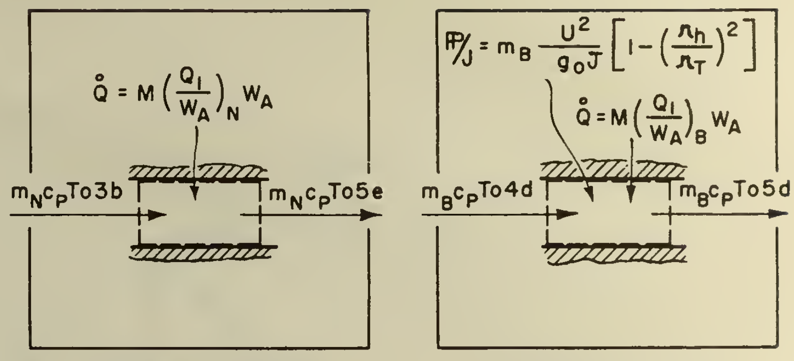


(a) FORCED CONVECTION AIR COOLED BLADES



(b) TRANSPIRATION AIR COOLED BLADES

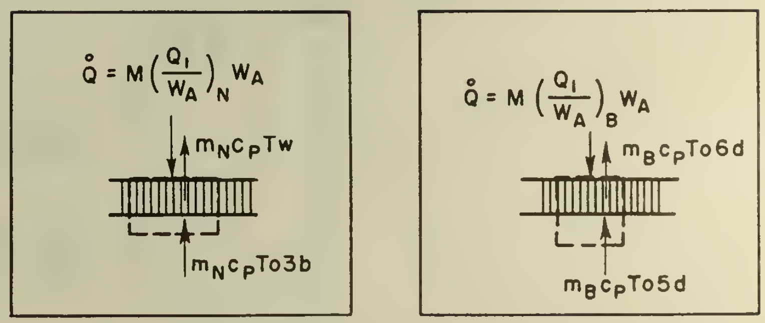
FIG. 3 – TYPICAL CYCLE DIAGRAMS FOR MAIN WORKING FLUID, M


$\mathring{Q} = M\left(\frac{Q_1}{W_A}\right)_N W_A$
$m_N c_p T_{o3b}$
$m_N c_p T_{o5e}$
$P/J = m_B \frac{U^2}{g_o J}\left[1-\left(\frac{\eta_h}{\eta_T}\right)^2\right]$
$\mathring{Q} = M\left(\frac{Q_1}{W_A}\right)_B W_A$
$m_B c_p T_{o4d}$
$m_B c_p T_{o5d}$

NOZZLE COOLANT PASSAGES

BLADE COOLANT PASSAGES

(a) FORCED CONVECTION BLADE COOLING

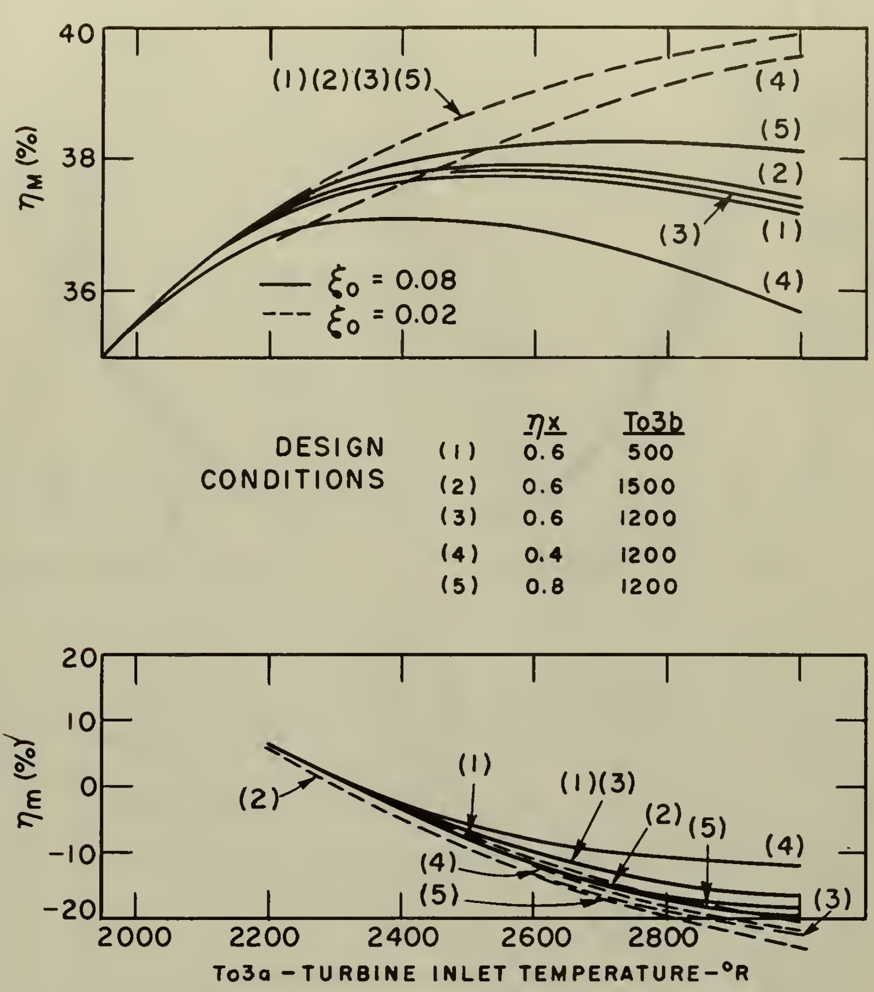

$\mathring{Q} = M\left(\frac{Q_1}{W_A}\right)_N W_A$
$m_N c_p T_w$
$m_N c_p T_{o3b}$
$\mathring{Q} = M\left(\frac{Q_1}{W_A}\right)_B W_A$
$m_B c_p T_{o6d}$
$m_B c_p T_{o5d}$

NOZZLE POROUS WALLS

BUCKET POROUS WALLS

(b) TRANSPIRATION BLADE COOLING

FIG. 4 – ENERGY BALANCES APPLIED TO COOLING PROCESSES



DESIGN CONDITIONS		ηx	To3b
	(1)	0.6	500
	(2)	0.6	1500
	(3)	0.6	1200
	(4)	0.4	1200
	(5)	0.8	1200

FIG. 5 - VARIATION IN η_M AND η_m WITH TURBINE INLET TEMPERATURE AND SYSTEM DESIGN PARAMETERS FOR FORCED CONVECTION AIR COOLED POWER PLANT

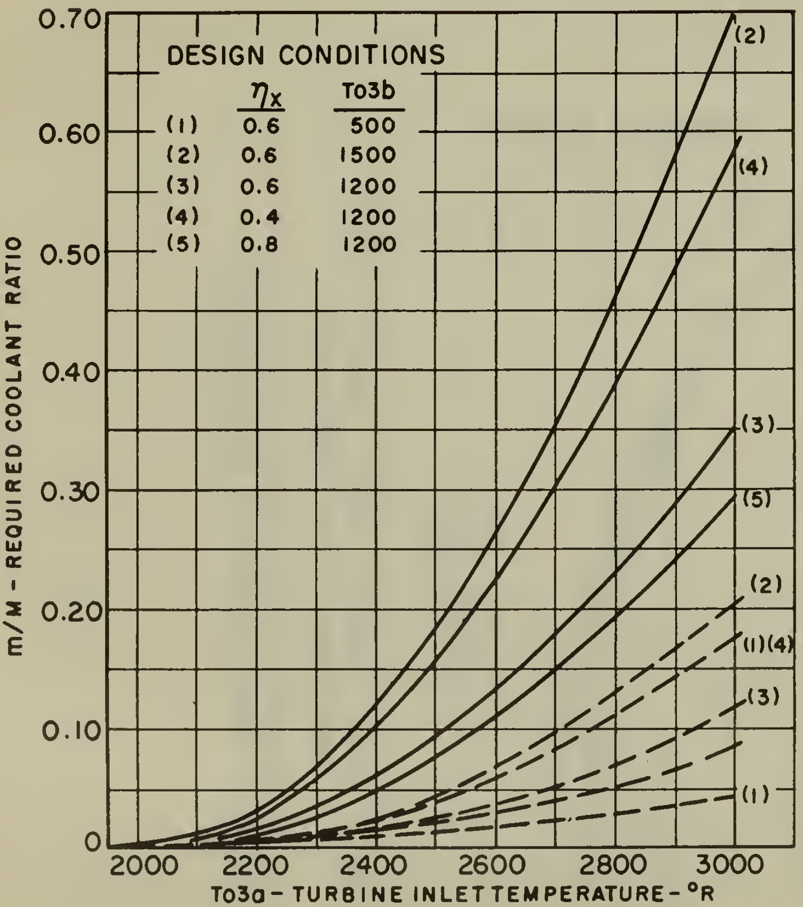


FIG. 6 - EFFECT OF TURBINE INLET TEMPERATURE AND DESIGN PARAMETERS ON REQUIRED COOLANT RATIO FOR FORCED CONVECTION ON AIR-COOLED POWER PLANT

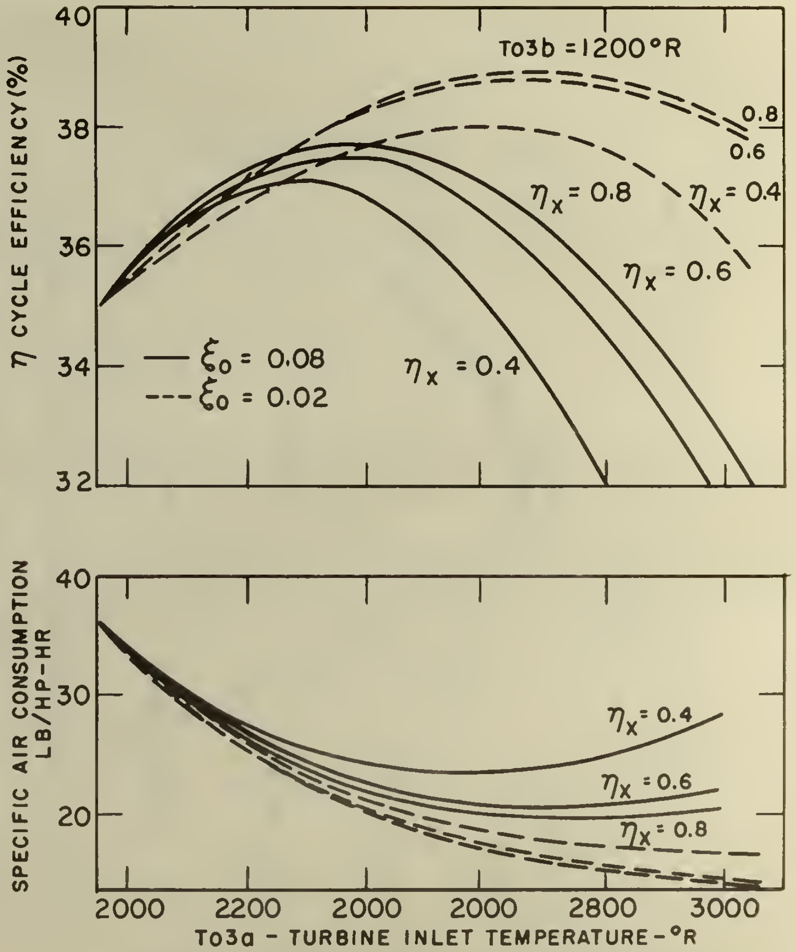


FIG. 7 - EFFECT OF BLADE COOLANT PASSAGE EFFECTIVENESS ON PERFORMANCE OF FORCED CONVECTION AIR COOLED POWER PLANT

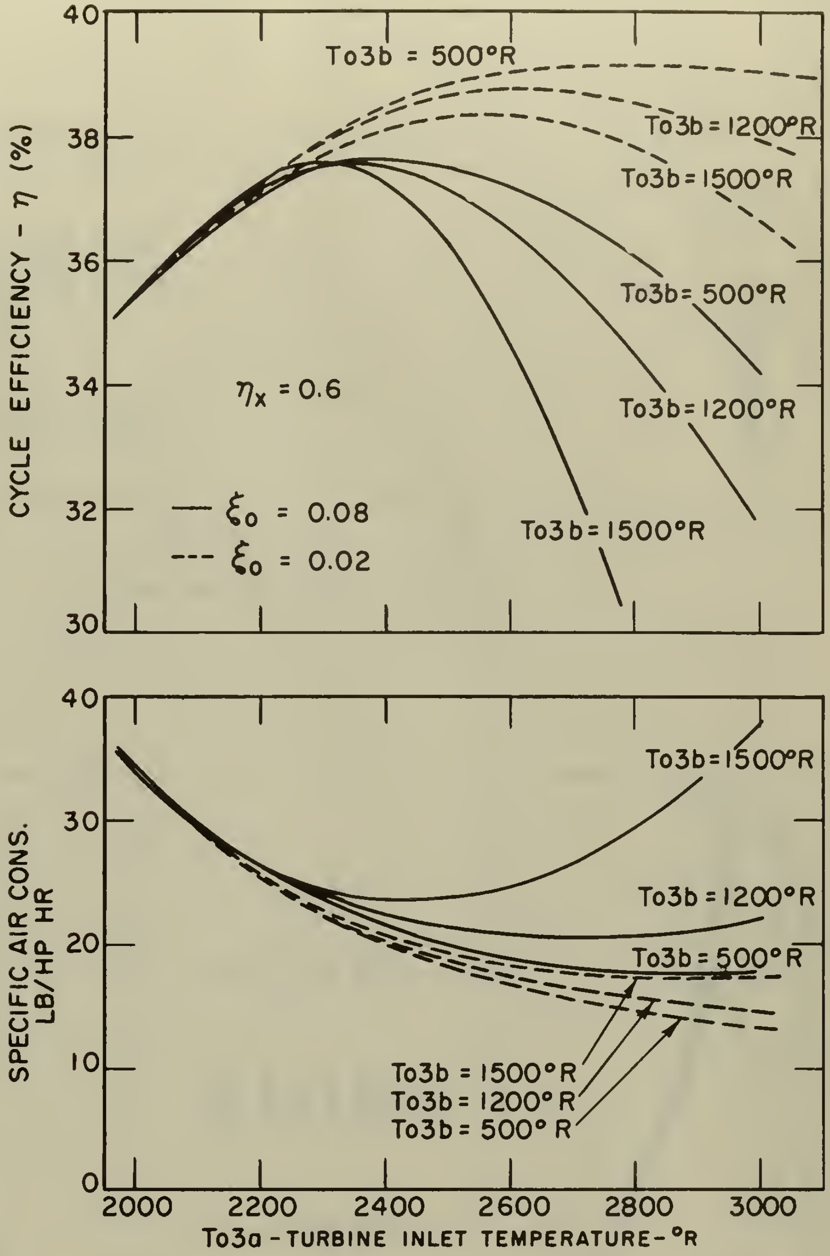


FIG. 8 - EFFECT OF COOLANT INLET TEMPERATURE ON PERFORMANCE OF FORCED CONVECTION COOLED POWER PLANT

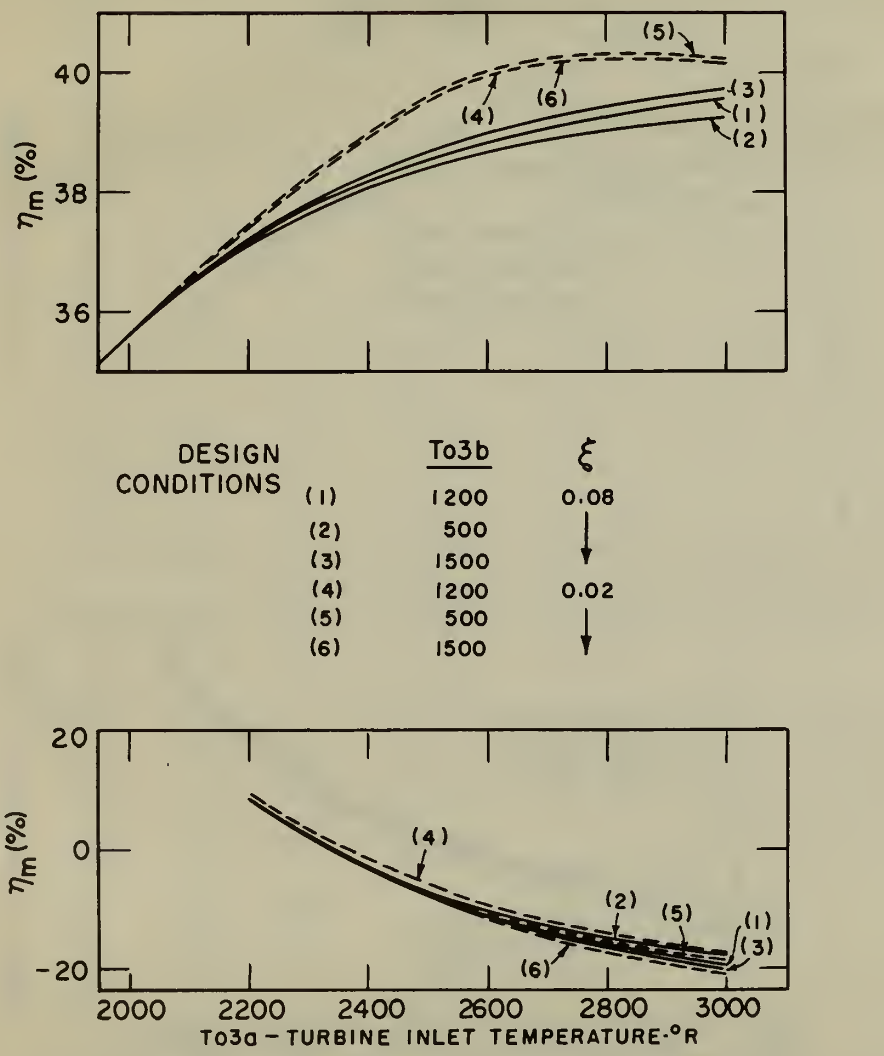


FIG. 9 - VARIATION IN η_M AND η_m WITH TURBINE INLET TEMPERATURE AND SYSTEM DESIGN PARAMETERS FOR TRANSPIRATION AIR COOLED POWER PLANT

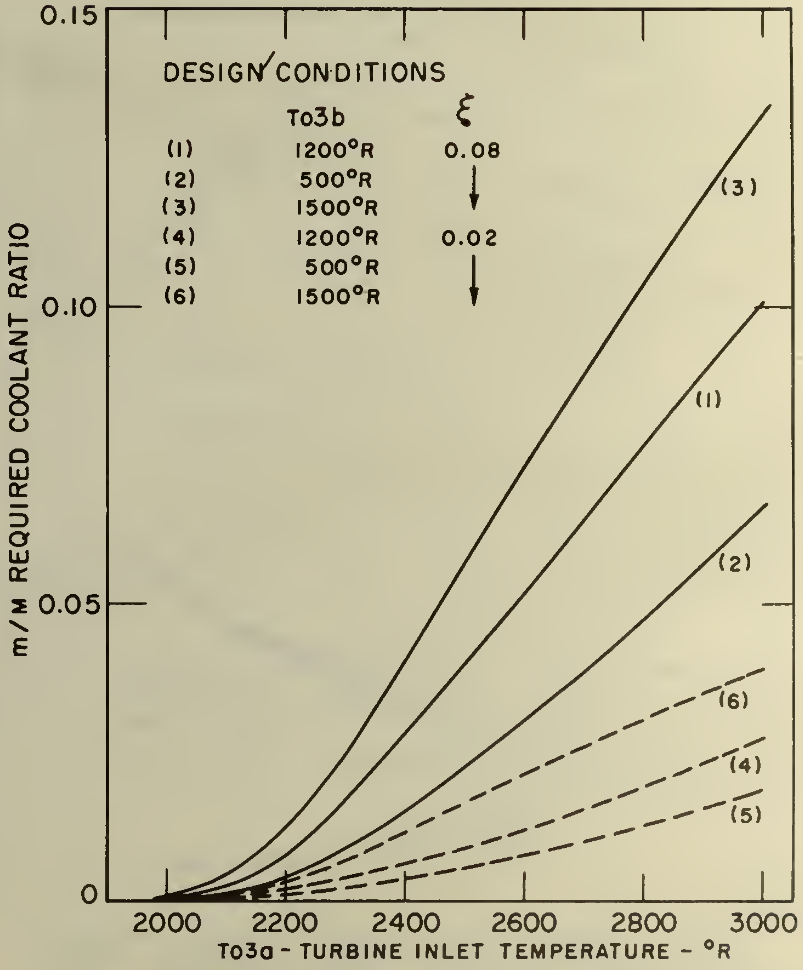


FIG. 10 - EFFECT OF TURBINE INLET TEMPERATURE AND DESIGN PARAMETERS ON REQUIRED COOLANT

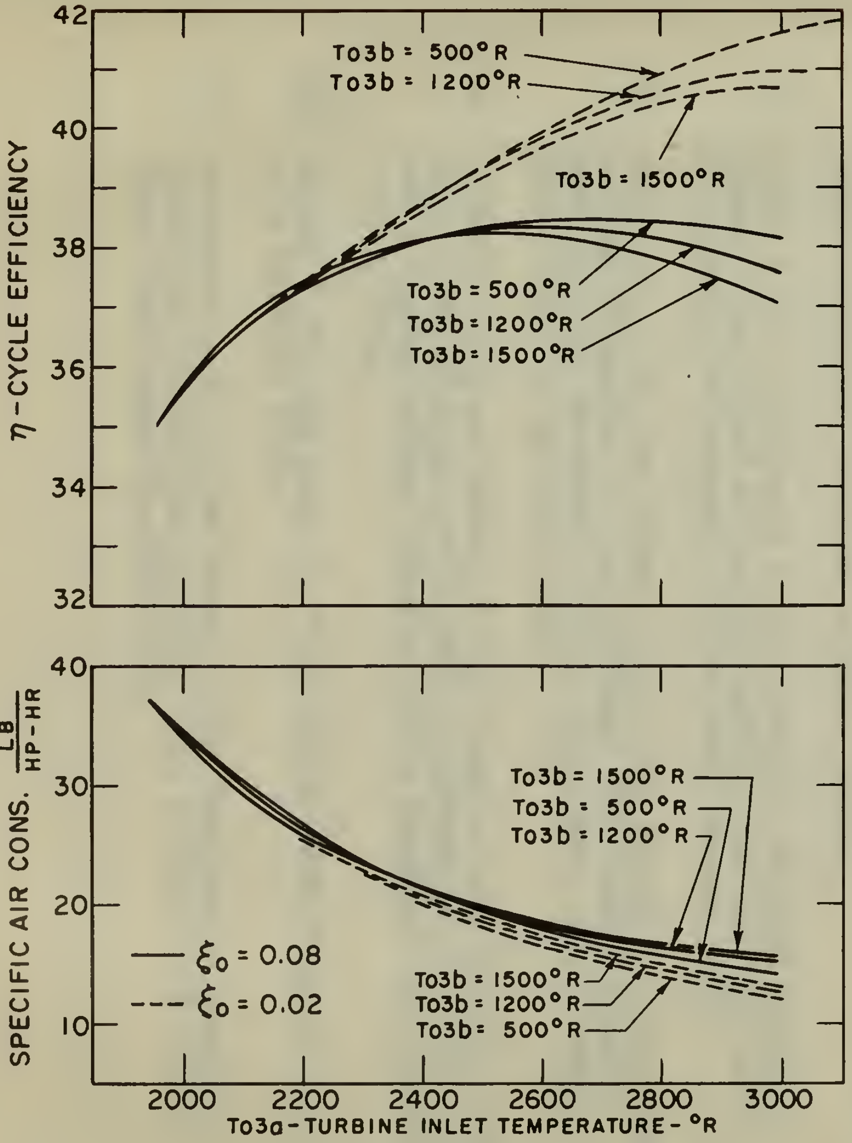


FIG. 11 - EFFECT OF BLADE COOLANT INLET TEMPERATURE ON PERFORMANCE OF TRANSPIRATION COOLED POWER PLANT

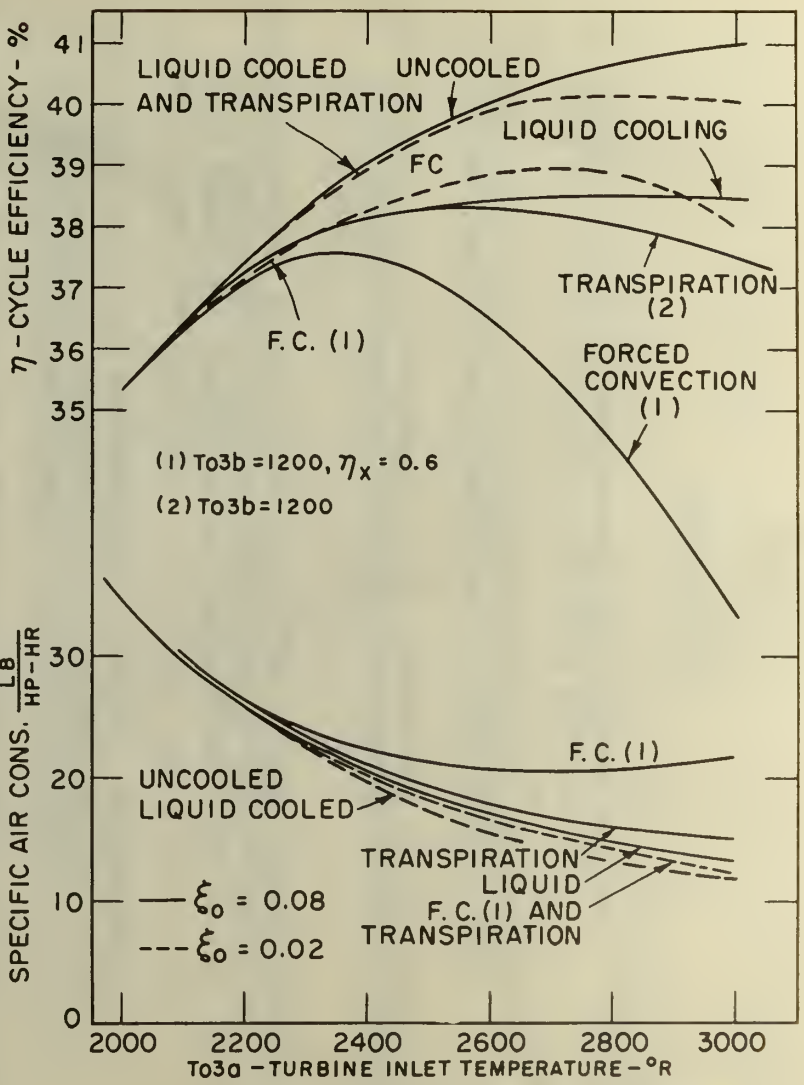


FIG. 12 - COMPARISON OF PERFORMANCE OF POWER PLANTS EMPLOYING DIFFERENT SYSTEMS OF COOLING.

APPENDIX A

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APPENDIX B

TABLE OF NOMENCLATURE

1. Symbols

$A_t = (A_t)_N = (A_t)_B$ - total transpiration area for blades, ft^2

C_p - specific heat at constant pressure, $btu/lb_m{-}^{\circ}R$

C_{p_1} - average for cooled turbine - .283, $btu/lb_m{-}^{\circ}R$

C_{p_2} - average for mixing process - .277, $btu/lb_m{-}^{\circ}R$

C_{p_3} - average for coolant in blades - .264, $btu/lb_m{-}^{\circ}R$

$De \equiv \frac{4S}{P}$ - equivalent diameter of blade coolant passages, ft

$f \equiv 2g_o \; {}_{o}\!/ \; V^2$ - friction factor

$G_o \equiv \frac{m_N}{S}$ or $\frac{m_B}{S}$ - coolant flow per unit area, lb_m/ft^2-sec

$G_t \equiv \frac{m_N}{A_t}$ or $\frac{m_B}{A_t}$ - coolant flow per unit area, lb_m/ft^2-sec

K_1 - impulse blade factor (defined in reference 18)

$k_1 \equiv M/S$ - turbine constant, lb_m/ft^2-sec

k - thermal conductivity, $btu/sec{-}ft{-}^{\circ}R$

ℓ - blade length, ft

M - main working fluid mass flow, lb_m/sec

$m = m_N + m_B$ - total coolant mass flow, lb_m/sec

$P = (P)_N = (P)_B$ - total perimeter of coolant passages, ft

p - pressure, lbf/ft^2

$\mathcal{P}$ - power, ft-lbf/sec

$\mathcal{P}_B$ - power output of turbine B, ft-lbf/sec

$\dot{Q}$ - heat flow, btu/sec

Q - heat flow per unit mass to infinitesimal blade row, btu/lb_m

Q_l - heat loss to blades per unit mass of M, btu/lb_m

Q_{in} - powerplant heat input per unit mass of M, btu/lb_m

R_A - reheat factor for Turbine A

R_B - reheat factor for Turbine B

$r \equiv \frac{P_{o3a}}{P_{o8}}$ - total power plant pressure ratio

$r_A \equiv \frac{P_{o3a}}{P_{o4a}}$ - expansion ratio turbine A

$r_B \equiv \frac{P_{o4a}}{P_{o8}}$ - expansion ratio turbine B

r_h - turbine wheel blade root radius, ft

r_T - turbine wheel blade tip radius, ft

$S = (S)_N = (S)_B$ - total crossectional flow area for blades, ft^2

T_w - blade wall temperature, ^oR

u - turbine wheel tip velocity, ft/sec

$\bar{u} \equiv u(\frac{r_h + r_T}{2})$ - mean blade velocity, ft/sec

V_t - tangential velocity, ft/sec

V_{rel} - velocity relative to rotor coolant passages, ft/sec

W - maximum work for infinitesimal stage, btu/lb_m

$W_A = \frac{\quad}{JM}$ - work output turbine A, btu/lb_m

$W_B = \frac{B}{(M + m)_J}$ - work output turbine B, btu/lb_m

$W_{C_1} \equiv {}_{C_1}/MJ$ - work input main compressor, btu/lb_m

$W_{net} \equiv \frac{net}{MJ}$ - net powerplant work output, btu/lb_m

ρ - density, lb_m/ft^3

η - powerplant cycle efficiency

C_1 - main compressor efficiency

ξ_o - loss factor (defined in reference 18)

τ - torque, lbf-ft

ω - angular velocity, rad/sec

2. Subscripts

o - refers to total stagnation conditions

o - refers to relative stagnation conditions

s - refers to constant entropy process

N - refers to nozzles

B - refers to blades

$*$ - denotes zero transpiration mass flow

1a-4a - refers to states of main working fluid

1b-3b - refers to states of total coolant

4d-6d - refers to states of bucket coolant

4e-6e - refers to states of nozzle coolant

7, 8 - refers to states of total mixture

APPENDIX C

TABLE OF POWERPLANT CONSTANTS

η_P	- Turbine polytropic efficiency	.88
η_{C_1}	- Main compressor efficiency	.85
η_{C_2}	- Auxiliary compressor efficiency	.85
r	- Total pressure ratio	15
u	- Turbine blade tip speed	1000 ft/sec
T_{o7}	- Inlet temperature of turbine B	1960°R
T_w	- Maximum allowable blade temperature	1960°R
P_{o1a}	- Main compressor inlet pressure	2118 psf
P_{o8}	- Powerplant exhaust pressure	2118 psf
T_{o1a}	- Main compressor inlet temperature	520°R
ξ_o	- Loss factor	0.02-0.08
K_1	- Impulse blade factor	4.0
r_h/r_T	- Ratio of turbine blade root to tip radius	0.7
k_1	- Turbine flow constant $\equiv M/S$	500 lb_m/ft^2-sec

APPENDIX D

CALCULATION METHOD FOR η_M

In accordance with Equation (4), η_M is expressed as:

$$\eta_M = \frac{W_A + W_B - W_{C_1}}{Q_{in}}$$

By use of Table 1, reference 10, it is possible to determine W_{C_1} and Q_{in} as follows.

$$W_{C_1} = \frac{h_{o2as} - h_{o1a}}{\eta_{C_1}} = 170.8 \text{ btu/lbm} \qquad (1\text{-d})$$

$$Q_{in} = h_{o3a} - h_{o2a} = h_{o3a} - 295.1 \text{ btu/lbm} \qquad (2\text{-d})$$

Reference 14 shows that the work output of turbines composed of an infinite number of infinitesimal stages may be calculated using the reheat factors given below.

$$W_A = \eta_P R_A (h_{o3a} - h_{o4as}) \qquad (3\text{-d})$$

$$W_B = \eta_P R_B (h_{o7} - h_{o8s}) \qquad (4\text{-d})$$

where

$$R_A{}^{(1)} = \frac{1 - \left(\frac{1}{r_A}\right)^{\eta_P (1 + Q_1/W_A)\frac{K-1}{K}}}{{}_P(1 + Q_1/W_A)\ 1 - \left(\frac{1}{r_A}\right)^{\frac{K-1}{K}}} \qquad (5\text{-d})$$

$$R_B{}^{(1)} = \frac{1 - (1/r_B)^{\eta_P \frac{K-1}{K}}}{{}_P\ 1 - \left(\frac{1}{r_B}\right)^{\frac{K-1}{K}}} \qquad (6\text{-d})$$

---

(1) In evaluating R_A and R_B, the following average values of k were employed:

k(cooled turbine) = 1.32
k(uncooled turbine) = 1.35

Also, W_A may be determined independently by reference to Fig. 3.

a) For a forced convection cooling system

$$W_A = \frac{h_{o3a} - h_{o7}}{1 + Q_1/W_A + \Delta H/W_A} \qquad (7\text{-}d)$$

b) For a transpiration air cooling system or liquid cooling

$$W_A = \frac{h_{o3a} - h_{o7}}{1 + Q_1/W_A} \qquad (8\text{-}d)$$

Now, having once determined Q_1/W_A and $\Delta H/W_A$, it is possible to calculate r_A by "trial and error" from Equations (3-d), (5-d), (7-d) (or (8-d) and the Gas Tables (reference 10). Since $r_B = r/r_A$, W_B can, at this point, be evaluated from Equations (6-d) and (4-d). A sample calculation employing this method is given in reference 18.

APPENDIX E

DERIVATION OF EQUATION (6)

Hawthorne (8) suggests the following approximate analysis for determining the heat transfer per lb of fluid flowing across a turbine blade row.

The Reynolds' analogy relating momentum transfer and heat transfer is

$$\frac{\alpha}{C_p G} = \frac{f}{2} \qquad (1\text{-e})$$

An energy balance for an element of length dx along the blade surface in the direction of flow is

$$dq = \alpha P(T_w - T_o)dx = S\ G\ C_p dT_o \qquad (2\text{-e})$$

Substituting Equation (1-e) into Equation (2-e) and integrating between points 1 and 2 at inlet and outlet of the blade passage results in

$$\frac{T_{o_1} - T_{o_2}}{T_{o_2}} = \frac{T_{o_2} - T_w}{T_{o_2}} \left(e^{\int_{x=o}^{L} \frac{fP}{2S} dx} - 1 \right) \qquad (3\text{-e})$$

Defining a loss factor ξ_o by

$$\xi_o = \int_{x=o}^{L} f \frac{P}{S} dx \qquad (4\text{-e})$$

Then since the heat transferred per lb of fluid is

$$Q = C_p \left(T_{o_1} - T_{o_2}\right) \qquad (5\text{-e})$$

Equation (3-e) with Equations (4-e) and (5-e) become

$$Q = C_p \left(T_{o_2} - T_w\right) \left(e^{\xi_o/2} - 1\right) \qquad (6\text{-e})$$

The maximum work per stage of infinite staged turbine or best efficiency of a single stage turbine is obtained when the magnitude of the term

$$\frac{W}{u^2/2g_oJ} = K_1 \qquad (7\text{-e})$$

is approximately $K_1 = 4$ for impulse blading and $K_1 = 2$ for reaction blading (reference 14). Actual values will be slightly less than these.

Then dividing Equation (6-e) by Equation (7-e), an expression for Q/W for a cascade of blades can be written as

$$\frac{Q}{W} = \frac{C_p(T_{o_2} - T_w)\,(e^{\xi_o/2} - 1)}{K_1 u^2/2g_oJ} \qquad (8\text{-e})$$

which is Equation (6).

Initial calculations for this study were made with a loss coefficient in the range $0.05 < \xi_o < 0.08$. Because of the varying opinions as to the correct value of ξ_o and because of its pronounced effect on cooled turbine performance, the range was extended to include $0.02 < \xi_o < 0.08$. It is believed that this range will adequately bracket the actual value of ξ_o for any particular blade design. It will also bring the data into closer agreement with more recent work completed on profile loss factors as mentioned below.

A discussion on the determination of ξ_o is presented here in order to show some of the reasoning behind the variance of opinion and to verify, if possible, the use of a loss coefficient in the range $0.02 < \xi_o < 0.08$.

For low Mach number and for the case in which (P/S) does not vary along the flow path, Hawthorne (reference 8) shows that $\xi_o = \xi_p$ the profile loss factor. This would correspond to the case of impulse blading. For reaction blading and nozzles having decreasing area in the direction of gas flow and for values of Mach number greater than zero, ξ_p becomes less than ξ_o.

Hawthorne (reference 8) combined the following equation for the heat transfer coefficient from gas to blade

$$Nu = 0.14\ (Re)^{0.68}\ (Pr)^{1/3} \qquad (9\text{-}e)$$

with the heat balance equation for a blade passage

$$\alpha\ S\ (T_w - T_o)\ dx = A \rho V\ C_p\ dT_o \qquad (10\text{-}e)$$

and Reynolds' analogy to obtain

$$(e^{\xi_o/2} - 1) = 0.14\ (Re)^{-.32}(Pr)^{-.67}\gamma \qquad (11\text{-}e)$$

where

$$\gamma = \frac{S}{A} = \frac{\text{heat transfer surface}}{\text{cross sectional flow area}}$$

Equation (11-e) yields $\xi_o = 0.026 - 0.084$ for Reynolds' number of $5 \times 10^5 > Re > 2 \times 10^4$ where γ varies from $5 < \gamma < 10$ for most blade shapes.

Subsequent to Hawthorne's analysis, Van Le (reference 18) completed a study of loss coefficients $\xi_p = f(i, \alpha_1/\alpha_2, \theta, Re, S/C)$. Data taken from charts in reference 18 show the following trends:

a) ξ_p decreases as α_1/α_2, Re, and S/C increase.

b) ξ_p increases as θ and i increase.

c) ξ_p for reaction blades in less than ξ_p for impulse blades.

d) For normal variations in turning angle θ, degree of reaction, α_1/α_2, pitch chord ratio, S/C, incidence angle i, and blade Reynolds' number, Re, the profile loss coefficient was found to vary within the limits $0.02 < \xi_p < 0.14$.

These values of ξ_p would seem to agree fairly well with Hawthorne's analysis that $\xi_o = \xi_p$ particularly at the higher values of Reynolds' number.

Smith and Pearson (reference 17) define a heat extraction coefficient, j, for a turbine stage

$$j = \frac{\Delta H}{W} \frac{u^2}{JgC_p(T_o - T_w)} \qquad (12\text{-e})$$

By comparison with Equation (8-e) for a cascade of blades, where $K_1 = 4$ for impulse blades, the following relation can be established

$$j = (e^{\xi_o/2} - 1) \cong \xi_o/2 \qquad (13\text{-e})$$

Reference 17 includes plots of j versus flow coefficient which show that j varies between $0.01 < j < 0.03$. These values show good agreement with Hawthorne's values for $\xi_o/2$ between $0.012 < \xi_o/2 < 0.042$.

Hubbart of NACA (reference 9) and Brown (reference 2) suggest the use of

$$Nu = .0296\ (Re)^{.8}$$

instead of Equation (9-e). Use of this equation results in values of ξ_o which are about ten percent less than those determined by Hawthorne.

APPENDIX F

ANALYSIS OF COOLANT FLOW IN ROTOR AND BLADES

-- FORCED CONVECTION AIR COOLED POWERPLANT

1. Analysis of bucket coolant flow

a. General -- flow in rotating coolant passages with heat transfer

Consider a stationary annular control volume inclosing the rotor from r to r + dr (see Fig. F-1). Applying the steady flow energy equation to the coolant within the control volume

$$\frac{d\mathbb{P}}{J} + d\dot{Q} = m_B\, C_p dT_o$$

From the theorem of moment of momentum (assuming slip factor = 1.0).

$$\frac{d\mathbb{P}}{J} = \frac{2m_B\omega^2}{g_o J}\, r dr$$

Therefore

$$m_B C_p dT_o = d\dot{Q} + \frac{2m_B\omega^2}{g_o J}\, r\, dr \tag{1-f}$$

At any radius r the relative (to coolant passage wall) stagnation temperature and total stagnation temperature may be written:

$$T_o = T + \frac{1}{2}\left[\frac{(V_{rel})^2}{C_p J g_o} + \frac{r^2\omega^2}{C_p J g_o}\right]$$

$$T_{or} = T + \frac{1}{2}\,\frac{(V_{rel})^2}{C_p J g_o}$$

Therefore

$$dT_{or} = dT_o - \frac{\omega^2}{C_p J g_o}\; r dr$$

Equation (1-f) can now also be written in terms of relative temperatures

$$m_B C_p dT_{or} = d\dot{Q} + \frac{m_B \omega^2}{g_o J}\; r dr \qquad (2\text{-f})$$

b. Flow in rotor (3-d - 4-d)

Assuming adiabatic flow in the rotor, Equations (1-f) and (2-f) are integrated to show

$$T_{or4d} = T_{o3b} + \frac{1}{2}\;\frac{u^2}{C_p J g_o}\;\left(\frac{r_h}{r_T}\right)^2 \qquad (3\text{-f})$$

$$T_{o4d} = T_{o3b} + \frac{u^2}{C_p J g_o}\left(\frac{r_h}{r_T}\right)^2 \qquad (4\text{-f})$$

c. Flow in buckets (4-d - 5-d)

Integrating (2-f) from (4-d) to (5-d)

$$\dot{Q} = M(Q_1/W_A)_B W_A = m_B C_p (T_{or5d} - T_{or4d}) - \frac{m_B \omega^2}{2 g_o J}\,(r_T{}^2 - r_h{}^2)$$

and

$$(m/M)_B = \frac{(Q_1/W_A)_B\; W_A}{C_{p_3}(T_{or5d} - T_{or4d}) - \frac{1}{2}\,\frac{u^2}{g_o J}\left[1 - (r_h/r_T)^2\right]}$$

Defining an apparent effectiveness for the bucket coolant passages as:

$$\eta_x{}' \equiv \frac{T_{or5d} - T_{or4d}}{T_w - T_{or4d}} \qquad (5\text{-f})$$

$$(m/M)_B = \frac{(Q_1/W_A)_B\, W_A}{C_{p_g}\left\{ \eta_x{}' \left[T_w - T_{o3b} - \frac{1}{2}\frac{u^2}{C_pJg}\left(\frac{r_h}{r_T}\right)^2 \right] - \frac{1}{2}\frac{u^2}{C_pJg}\left[1 - \left(\frac{r_h}{r_T}\right)^2\right]\right\}} \qquad (6\text{-}f)$$

d. Relation of $\eta_x{}'$ to coolant flow conditions

Combining Newton's law of cooling with Equation (2-f)

$$d\dot{Q} = \alpha P'(T_w - T_{or})dr = m_B C_p T_{or} - m_B \frac{\omega^2}{g_o J} rdr \qquad (7\text{-}f)$$

Assuming $\alpha P'/m_B C_p$ and T_w do not vary with r the above equation is integrated to yield

$$\frac{T_w - T_{or5d}}{T_w - T_{or4d}} = e^{-\left(\frac{4}{G_oC_p}\ \frac{\ell}{De} + \beta\right)} \qquad (8\text{-}f)$$

where

$$\beta = \frac{\omega^2}{C_pJg_o}\int_{r_h}^{r_T} \frac{rdr}{T_w - T_{or}}$$

β is later integrated assuming a linear variation of T_{or} with radius, however, for the moment, comparing (8-f) with (5-f) it is seen that

$$\eta_x{}' = 1 - e^{-\left(\frac{4}{G_oC_p}\ \frac{\ell}{De} + \beta\right)} \qquad (9\text{-}f)$$

Neglecting the effect of free convection on the heat transfer process within the blade coolant passage, the Stanton number is expressed by the empirical pipe flow equation (reference 15).

$$\frac{\alpha}{G_cC_p} = .023\left(\frac{G_cDe}{u}\right)^{-.2}\left(\frac{C_pu}{k}\right)^{-.67}$$

The same reference relates the friction factor to the Reynolds' number (for $5000 < G_c De/\mu < 200{,}000$) as follows:

$$f = .046 \left(\frac{G_c De}{\mu}\right)^{-.2} \tag{11-f}$$

Assuming that $C_p \mu/k \approx .69$, Equations (9-f), (10-f), and (11-f) result in

$$\eta_x{}' = 1 - e^{-(2.56\ f\ \ell/De + \beta)} \tag{12-f}$$

2. Analysis of nozzle coolant flow

The equations corresponding to (1-f) through (12-f) can readily be attained for the flow through the non-rotating nozzle coolant passages by equating $\omega = 0$. The pertinent results are listed below.

$$T_{o4e} = T_{or4e} = T_{o3b} \tag{13-f}$$

$$(m/M)_N = \frac{(Q_1/W_A)_N\ W_A}{C_{p_g} \eta_x (T_w - T_{o3b})} \tag{14-f}$$

$$\eta_x = 1 - e^{-2.56\ f\ \ell/De} \tag{15-f}$$

3. Evaluation of $\eta_x{}'$

The nozzles and buckets in this powerplant are assumed to have identical coolant passage construction. The coolant flows for nozzles and buckets will certainly be of at least the same order of magnitude. Therefore, it is assumed that the term, $f\ \ell/De$, is very nearly the same for both. Comparing Equations (12-f) and (15-f)

$$\eta_x{}' = 1 - (1 - \eta_x)\, e^{-\beta} \tag{16-f}$$

Now, returning to the definition of β (Section 1), and performing the integration, assuming that T_{or} is linear with r

$$\beta = E(F \ell n\, A - 1) \qquad (17\text{-}f)$$

where

$$E = \frac{\dfrac{u^2}{C_p J g_o} \;\; 1 - \dfrac{r_h}{r_T}^{\;2}}{\eta_x'\left[T_w - T_{o3b} - \dfrac{u^2}{2C_p J g_o}\left(\dfrac{r_h}{r_T}\right)^2\right]}$$

$$F = \frac{1}{\eta_x'} + \frac{r_h/r_T}{1 - r_h/r_T}$$

$$A = \frac{1}{1 - \eta_x'}$$

The combination of Equations (16-f) and (17-f) determines η_x' in terms of η_x, T_{o3b} and certain powerplant operating constants. In Fig. F-2, η_x' is plotted against η_x for the three values of T_{o3b} used in this investigation. The values of η_x' used in the calculations are tabulated below.

η_x'	η_x	T_{o3b}
.438	.4	1200°R
.630	.6	1200°R
.820	.8	1200°R
.617	.6	500°R
.650	.6	1500°R

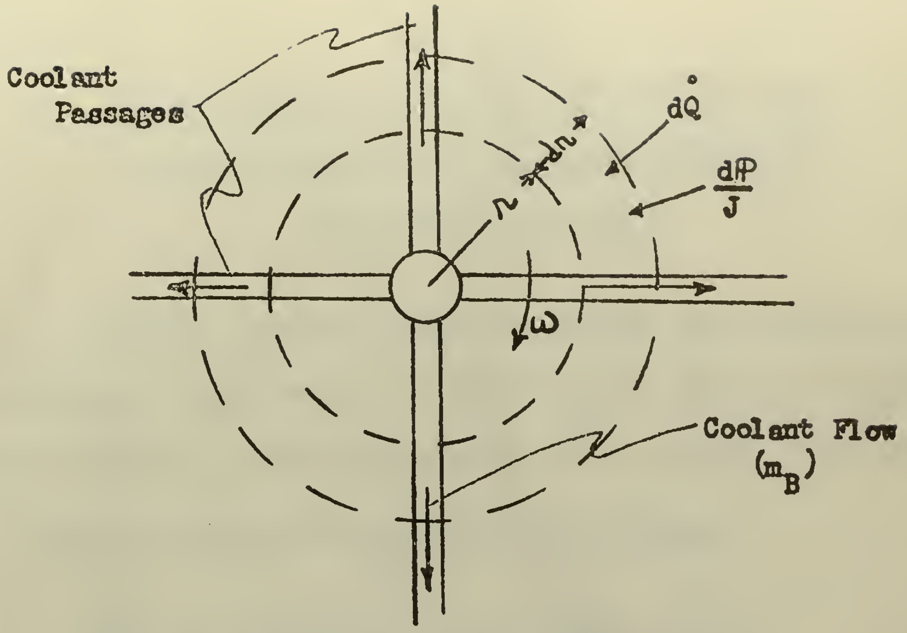

Fig. F-1 Control Volume for Coolant Analysis

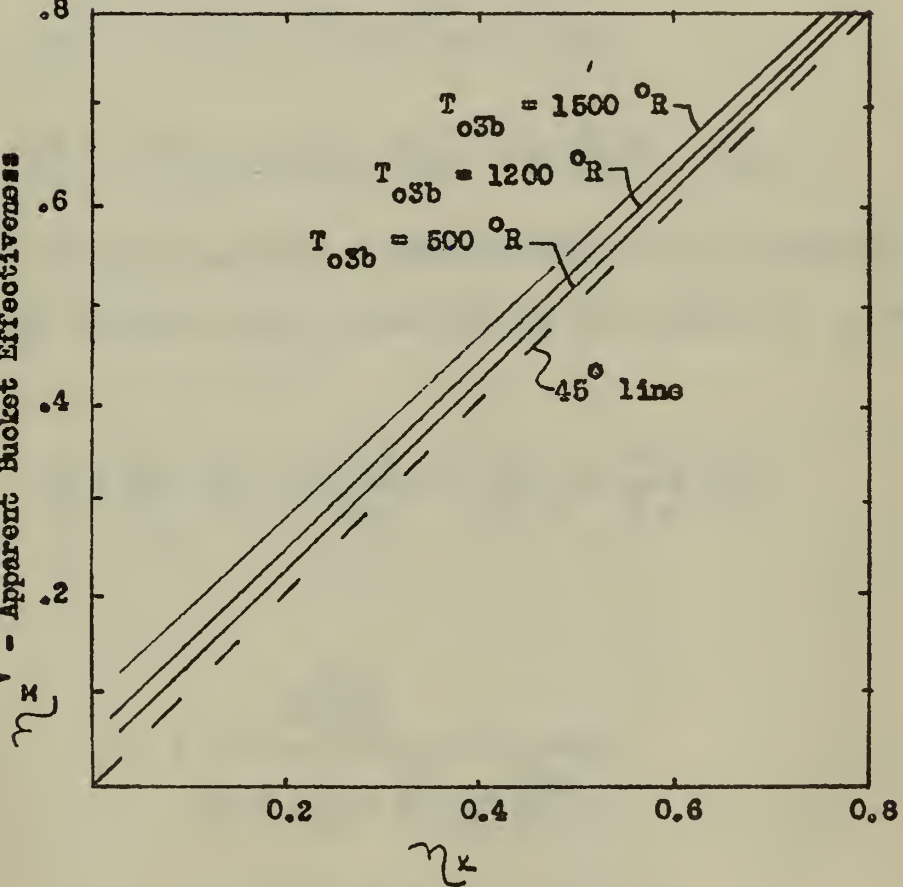

Fig. F-2 Plot of η_x' Vs. η_x

APPENDIX G

DERIVATION OF EXPRESSION FOR ENTHALPY LOSS DUE TO MIXING ($\Delta H/W_A$) IN FORCED CONVECTION AIR COOLED POWERPLANT

Assuming adiabatic mixing of coolant and working fluid, and neglecting variation in specific heat, the steady flow energy equation applied to the mixing chamber, F is written as follows:

$$m_N T_{o6e} + m_B T_{o6d} + M T_{o4a} = (m_N + m_B + M)\, T_{o7} \qquad (1\text{-}g)$$

But, by definition, $\Delta H \equiv C_{p_2}\,(T_{o4a} - T_{o7})$

and also

$$T_{o7} = T_w;\quad T_{o6e} = T_{o5e};\ T_{o6d} = T_{o5d}$$

$$\therefore \qquad \frac{\Delta H}{W_A} = \frac{C_{p_2}}{W_A}\left[(m/M)_N(T_w - T_{o5e}) + (m/M)_B(T_w - T_{o5d})\right] \qquad (2\text{-}g)$$

The above equation is further modified by introducing the expressions for the coolant ratios and the definitions of $_x$ and $_{x'}$.

$$\frac{\Delta H}{W_A} = \frac{C_{p_2}}{C_{p_3}}\ \left(\frac{Q_1}{W_A}\right)_N \left(\frac{1 - \eta_x}{\eta_x}\right) + \left(\frac{Q_1}{W_A}\right)_B \left(\frac{1 - \eta_{x'} - \epsilon}{\eta_{x'} - \sigma}\right) \qquad (3\text{-}g)$$

where

$$\epsilon \ = \frac{\dfrac{u^2}{2C_p J g_o}}{T_w - T_{o3b} - \dfrac{u^2}{2C_p J g_o}\left(\dfrac{r_h}{r_T}\right)^2}$$

$$\sigma = \frac{\frac{u^2}{2C_p J g_o}\left[1 - (\frac{r_h}{r_T})^2\right]}{T_w - T_{o3b} - \frac{u^2}{2C_p J g_o}\,(r_h/r_T)^2}$$

Values of ϵ and σ used in this investigation are tabulated below:

T_{o3b}	ϵ	σ
500	.0438	.0253
1200	.096	.0497
1500	.164	.0847

APPENDIX H

DERIVATION OF EQUATIONS FOR ROTOR PUMPING WORK

1. General analysis -- varying coolant flow in rotating passages

Consider stationary annular control volume which incloses rotor between r and r + dr (See Fig. F-1). For this derivation only, the following definitions exist

n ≡ mass flow within coolant passages at radius r

δ_n ≡ mass leaving coolant passages between r and r + dr (i.e., due to transpiration).

From the theorem of moment of momentum

Net external torque = net efflux of angular momentum from the control surface.

$$d\tau = \delta_n (r + \frac{dr}{2})^2 \ \frac{\omega}{g_o} (n + (\frac{dn}{dr}) \, dr)(r + dr)^2 \frac{\omega}{g_o} - \frac{r^2 \omega n}{g_o} \tag{1-h}$$

From the continuity equation

$$\delta_n = - (\frac{dn}{dr}) \, dr \tag{2-h}$$

Combining the two above equations, and neglecting terms containing $(dr)^2$ results in

$$d\tau = \frac{2n\omega r dr}{g_o} \tag{3-h}$$

2. Forced convection cooling: $n = m_B$ = constant

$$\tau = \frac{m_B r_T^2 \omega}{g_o}$$

$$W_p' = \frac{\mathcal{P}_p}{m_B J} = \frac{\tau\omega}{m_B J} = \frac{u^2}{g_o J}$$

$$W_p = \frac{\mathcal{P}_p}{mJ} = \frac{(m/M)_B}{m/M}\ \frac{u^2}{g_o J} \qquad (4\text{-h})$$

3. Transpiration cooling

$$0 < r < r_h \qquad n = m_B$$

$$r_h < r < r_T \qquad n = m_B\left(1 - \frac{r - r_h}{r_T - r_h}\right) \qquad (1)$$

Integrating Equation (3-h), using this variation in n results in

$$\tau = \frac{m_B \omega r_T^2}{g_o}\left[1 - \frac{2}{3}\left\{\frac{1 - (r_h/r_T)^3}{1 - r_h/r_T}\right\} + \frac{r_h}{r_T} + \left(\frac{r_h}{r_T}\right)^2\right]$$

$$\tau = .730\ \frac{m_B \omega r_T^2}{g_o}$$

$$W_p = .730\ \frac{(m/M)_B}{(m/M)}\ \frac{u^2}{g_o J} \qquad (5\text{-h})$$

---

(1) Assuming that n varies linearly with r within the blades.

APPENDIX J

DERIVATION OF EQUATION (17)

Equation (19) (film theory) expresses the effect of mass transfer on the heat transfer coefficient as

$$\alpha/\alpha^* = \left(\frac{G_t C_p}{\alpha^*}\right)/\left(e^{\left(\frac{G_t C_p}{\alpha^*}\right)} - 1\right)$$

In reference 14, the heat transfer to a cascade of blades (no mass transfer present) is expressed

$$MQ_1^* = M\,C_p(T_o - T_w)\left(e^{\xi_o^*/2} - 1\right) \qquad (1\text{-}j)$$

Newton's law of cooling relates the heat transfer to the temperature difference

$$MQ_1^* = \alpha^* A_t (T_o - T_w) \qquad (2\text{-}j)$$

From Equations (1-j) and (2-j)

$$\frac{MC_p}{\alpha^* A_t} = \frac{1}{e^{\xi_o^*/2} - 1} \qquad (3\text{-}j)$$

Noting that for the nozzles, say

$$\left(\frac{MC_p}{\alpha^* A_t}\right)_N = \frac{G_t C_p}{(m/M)_N \alpha^*}$$

Therefore

$$\left(\frac{G_t C_p}{\alpha^*}\right)_N = (m/M)_N/\left(e^{\xi_o^*/2} - 1\right)$$

and

$$\frac{\alpha}{\alpha^*} = \frac{(m/M)_N/(e^{\xi_o^*/2} - 1)}{e^{\left[\frac{(m/M)_N}{e^{\xi_o^*/2} - 1}\right]} - 1} \tag{4-j}$$

Equation (4-j) could have equally well been written for the heat transfer to the buckets.

APPENDIX K

ANALYSIS OF WATER COOLED POWERPLANT

The efficiency of a cooled powerplant is, as seen from Equation (3), exactly equal to η_M if the quantity η_m is zero. For the water cooled powerplant studied, since the coolant does not mix with the working fluid:

$$\eta_m(\text{water cooled}) = - \frac{W_p + W_{C_2}}{Q_{in}} \qquad (1\text{-k})$$

The term W_p, rotor pumping work, is zero since the coolant flows both inward and outward in the rotor. Therefore, in order that η_m be zero, W_{C_2}, the work required to circulate the coolant must also be zero. This condition is fulfilled if the net centrifugal pressure rise of the coolant as it passes through the rotor and blades is equal to or greater than the net pressure loss in the remainder of the circuit. It is assumed that any excess pressure rise will be dissipated in throttling rather than converted into useful work.

The net pressure rise caused by the centrifugal field and density change is obtained by assuming the outward flowing coolant is all at density ρ_1 and the inward flow coolant at density ρ_2. (See F-1). Successive application of Bernoulli's equation yields:

$$\Delta p \text{ rise } 1 \rightarrow 2 = \frac{u^2}{2g_o}(\rho_1 - \rho_2) \approx \bar{\rho}\frac{u^2}{2g_o}\left[1 - \frac{\rho_2}{\rho_1}\right] \qquad (2\text{-k})$$

The total frictional pressure loss in the coolant circuit is approximated by multiplying the nozzle pressure drop by six. (Since the nozzles, buckets, rotor, stator, heat exchanger and coolant lines may be thought to cause approximately equal pressure losses).

$$\Delta P_{friction} = 6\ (\Delta P)_N = 24\ f\ \frac{\ell}{De}\ \frac{K_1^{\,2}(m/M)_N^{\,2}}{2\ \bar{\rho}\ g_o} \qquad (3\text{-}k)$$

From combination of Equations (2-k) and (3-k) it is seen that W_{C_2} is zero if

$$B = \frac{24\ f\ (\ell/De)\ K_1^{\,2}\ (m/M)_N^{\,2}}{\bar{\rho}^{\,2}\ u^2\left[1 - \rho_2/\rho_1\right]} < 1 \qquad (4\text{-}k)$$

The coolant ratio required for the nozzles is obtained by applying a heat balance to the nozzle coolant passages.

$$(m/M)_N = \frac{(Q_1/W_A)_N\ W_A}{C_p(T_w - T_2)\,\eta_{x\ell}} \qquad (5\text{-}k)$$

where

$$\eta_{x\ell} = \frac{T_3 - T_2}{T_w - T_2} \qquad (6\text{-}k)$$

The term $f\ \ell/De$ is obtained by an analysis similar to that carried out in Appendix F.

$$f\ \ell/De = \frac{1}{\phi}\ \ell n\ \left(\frac{1}{1-\eta_x}\right) \qquad (7\text{-}k)$$

where ϕ is a ratio of Prandtl numbers.

$$\phi = \frac{\left(\frac{C_p\mu}{k}\right)^{-.67}\ \text{at } T = \frac{T_3 + T_2}{2}}{\left(\frac{C_p\mu}{k}\right)^{-.67}\ \text{at } T = 510^{\circ}\ R} = \frac{\left(\frac{C_p\mu}{k}\right)^{-.67}\ \text{at } T = \frac{T_3 + T_2}{2}}{(.25)}$$

In order to prevent boiling in the coolant system, it is necessary to limit the temperature rise in the blades and hence the effectiveness. Assuming equal temperature rises in nozzles and buckets the maximum effectiveness as limited by T_3 is

$$\eta_{x\ell_{max}} = \frac{1}{2}\left(\frac{T_3 - T_1}{T_w - T_1}\right) \qquad (8\text{-}k)$$

In Fig. K-2, the above relation is plotted for the case where T_3 is limited to the boiling point of water at atmospheric pressure (T_3 = 670°R). Larger values of T_3 could, of course, be obtained if the system were pressurized.

B, the ratio of frictional pressure drop to centrifugal pressure rise is plotted in Fig. K-3 for two values of coolant inlet temperature, T_1. In this plot $\eta_{x\ell}$ is assumed equal to $\eta_{x\ell_{max}}$ as per K-2. It is seen that throughout the range of turbine inlet temperatures investigated, B is considerably less than unity. Hence, the required pumping work, W_{C_2}, is zero and η is just equal to η_M.

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